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LETTER TO THE EDITOR

The three-vertex in the closed half-string field theory and the general gluing and resmoothing theorem

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Abstract. In this letter we prove that the half-string three-vertex in closed string field theory satisfies the general gluing and resmoothing theorem. We also demonstrate how one calculates amplitudes in the half-string approach to closed string field theory, by working out explicitly a few simple three-amplitudes.

Motivated by the similarities between Yang–Mills theory and Witten's open string field theory [1], it was first suggested in [2, 3] and proved rigorously in [4–6], that physical open strings can be viewed as infinite-dimensional matrices. In particular; the open-string three-vertex can be represented as a trace [4–7]. This trace can be generalized to represent any N-string ($N \geqslant 3$) tree-level scattering amplitude [8]:

$$A_N = \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda_1 \ldots \mathrm{d}\lambda_N}{SL(2, \Re)} \operatorname{Tr}(\exp(\lambda_1 M) A_1 \ldots \exp(\lambda_N M) A_N) \tag{1}$$

where M is the generator of infinitesimal shifts of the mid-point of the string, so in fact one is shifting this point to every possible position.

An analogous construction for closed strings was formulated in [8, 9]. Using a functional approach [8], the analogue of equation (1) for closed strings was shown to give the correct dual amplitudes. The HS (half-string) operator formalism of a closed string [9] (here after referred as (I)) indicates that the restricted polyhedra of the classical non-polynomial string field theory, can be represented as traces of infinite-dimensional matrices, with operator insertions that reparametrize the half-strings. Also, in (I), the factorization of a closed string was established.

In this letter, we wish to discuss one further crucial property (which was not addressed in (I)) that one should expect of a correct formalism of CSFT (closed string field theory). Namely, the vertices of the theory must satisfy the GGR (general gluing and resmoothing) theorem of [10]. We will also give a few simple examples of how to calculate closed string amplitudes. Although some computational details are presented in this letter, this letter is not meant to be self-contained in the sense that we rely on (I) for notation and indeed for many other details only alluded to below.

We have seen that the closed HS vertices calculated in (I), i.e. $N = 1, 2$ and 3, do agree with the standard results for the closed bosonic string. However, we still have to

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compare HS higher-order vertices with other vertices appearing in the literature. In [10], LPP (LeClair, Peskin and Preitschopf) define the three-vertex through the relation

$$\langle V_{123}||A\rangle_1|B\rangle_2|C\rangle_3 = \langle T^2 h[\Theta_A]Th[\Theta_B]h[\Theta_C]\rangle$$

where Θ_A is the normal ordered operator that creates the state $|A\rangle$

$$|A\rangle = \Theta_A|0\rangle$$

and T is an $SL(2; C)$ transformation such that $T^3 = 1$. Higher-order vertices are defined analogously.

It is precisely these type of vertices that are used, as a starting point by Kugo and Suehiro, to construct the restricted polyhedra and to show that the resulting theory is gauge invariant [11].

Let us start by comparing the two-point vertex in both formalisms (i.e. the HS one and the standard one). By definition, the state $|V_{12}\rangle$ (or simply $|V^{(2)}\rangle$ in our language) imposes the condition $X_1(\sigma) = X_2(\pi - \sigma)$, which is equivalent to imposing that $\alpha_n^{(1)} = \alpha_{-n}^{(2)}$ at the level of operators. If we write the string coordinate in terms of the complex coordinate $z = \exp(\tau + \mathrm{i}\sigma)$ (take $\tau = 0$), then this condition is equivalent to imposing $X_1(z) = X_2(-1/z)$. This can be used to define the BPZ† (Belavin, Polyakov and Zamolodchikov) conjugate to $|A\rangle$ as

$$\langle A| = \langle 0|I[\Theta_A(0)]$$

where $I(z) = 1/z$ is a conformal map, i.e.

$$I[A(z, z^*)] = A'\left(\frac{1}{z}, \frac{1}{z^*}\right).$$

The (BPZ) inner product is defined as

$$\langle A|B\rangle = \langle 0|I[\Theta_A(0)]\Theta_B(0)|0\rangle = \langle V_{AB}||A\rangle|B\rangle.$$

It is easy to see that the two-vertex constructed with LPP's prescription using the I defined above, corresponds exactly to V_2 in the HS formalism.

A very important property that any vertex must satisfy is that the new vertex produced by the contraction of two other vertices by the conformal field theory inner product BPZ, is precisely the one which results from first sewing the corresponding two Riemann surfaces via the map I, and then constructing the vertex on that surface. In other words

$$\langle V^{(4)}_{ABEF}| = \langle V^{(3)}_{ABC}|\langle V^{(3)}_{DEF}||V^{(2)}_{CD}\rangle \tag{2}$$

with

$$\langle V_{1234}||A\rangle_1|B\rangle_2|C\rangle_3|D\rangle_4 = \langle T^2 h[\Theta_A]Th[\Theta_B]IT^2[\Theta_C]IT[\Theta_D]\rangle.$$

This is the generalized gluing and resmoothing (GGR) theorem for $N = 3$ [10].

It is not hard to show that the HS vertices constructed in (I) satisfy this theorem. The proof steams out from the fact that they are written as traces, and that the transformation from half-strings to full-strings is non-singular. To see this, denote HS string field matrices as $A_{nm} = \langle nm|\Lambda\rangle$‡ where the indices n and m refer to the left and right parts of the string. Completeness of the transformations means that Parseval's identity, $I = \sum_{nm} |n; m\rangle\langle n; m|$,

† See [12].

‡ Here $\langle\Lambda|$ represent a complete basis of string states.

works both ways, and $I = \int \mathrm{D}\Lambda|\Lambda)(\Lambda|$. The right-hand side of equation (2) can be written in the HS language (summing over repeated indices) as

$$V^{(3)}_{125}V^{(2)\dagger}_{56}V^{(3)}_{634} = \int \mathrm{D}\Lambda^5\mathrm{D}\Lambda^6\langle nm|\Lambda^1)\langle mk|\Lambda^2)\langle kn|\Lambda^5)$$
$$\times(\Lambda^5|pq\rangle(\Lambda^6|qp\rangle\langle rs|\Lambda^6)\langle sv|\Lambda^3)\langle vr|\Lambda^4).$$

Using (twice) that $\int \mathrm{D}\Lambda\langle kn|\Lambda)(\Lambda|pq\rangle = \delta_{kp}\delta_{nq}$ (orthogonality), we can write the above identity as

$$V^{(4)}_{1234} = \langle nm|\Lambda^1)\langle mk|\Lambda^2)\langle kq|\Lambda^3)\langle qn|\Lambda^4)$$

which is just the left-hand side of (2). This can be generalized to higher point vertices [13]. Notice that this result is valid for both open and closed strings.

In what follows we show how one calculates some simple three-string amplitudes. At tree level the ghosts decouple and the amplitude is given by $A_3 = \langle V_{123}|\Psi_1\rangle|\Psi_2\rangle|\Psi_3\rangle$, with $|\Psi_i\rangle$ the three external states satisfying the *physical conditions*

$$L_n|\Psi\rangle = \widetilde{L}_n|\Psi\rangle = 0 \qquad (L_0-1)|\Psi\rangle = (\widetilde{L}_0-1)|\Psi\rangle = 0 \qquad (L_0-\widetilde{L}_0)|\Psi\rangle = 0.$$

This can be expressed in terms of the closed-string vertex as

$$A_3 = \langle 0_{123}|N_0\exp\tfrac{1}{2}(\alpha^r_nN^{rs}_{nm}\alpha^s_m + \widetilde{\alpha}^r_nN^{rs\,*}_{nm}\widetilde{\alpha}^s_m)|\Psi_1\rangle|\Psi_2\rangle|\Psi_3\rangle.$$

The Neumann coefficients†, N^{rs}_{nm}, are related to the change of representation matrices between HS and FS (full-string) coordinates [9]. N_0 is the normalization; it is chosen such that the 3-tachyon amplitude is one. In general for closed strings $P^2 = (4/\alpha')(1-N) = 8(1-N)$ where $\alpha' = 1/2$ is our Regge slope convention and N is the number operator $(N = \widetilde{N})$. From the last equation one has

$$\langle V_3|N_0 = \langle 0_{123}|N_0\exp\left(-\ln\frac{3^3}{2^4}\left(3-\sum_{r=1}^{3}N_r\right) + \text{higher oscillators}\right)$$
$$= \langle 0_{123}|\exp\left(\ln\frac{3^3}{2^4}\left(\sum_{r=1}^{3}N_r\right) + \text{higher oscillators}\right).$$

For three tachyons $N_r = 0$; $r = 1, 2, 3$, and the above amplitude is one.

Now let us look at the (tachyon, tachyon, graviton) amplitude. The graviton corresponds to the first excited state $(N_G = 1)$ of the closed string; it has zero mass

$$|\Psi_G\rangle = G_{\mu,\nu}(P)\alpha^{3\mu}_{-1}\widetilde{\alpha}^{3\nu}_{-1}|0\rangle$$

where $G_{\mu,\nu}(P)$ is a symmetric traceless tensor. The physical conditions imply

$$P^\mu G_{\mu,\nu} = P^\nu G_{\mu,\nu} = 0.$$

If we label the two tachyon states by $i = 1, 2$ and the graviton by $i = 3$, then the amplitude is given by

$$A_3(T,T,G) = \langle 0_{123}|\exp\frac{1}{2}\left(\sum_{n,m\geqslant 1}\alpha^r_nN^{rs}_{nm}\alpha^s_m + \sum_{n,m\geqslant 1}\widetilde{\alpha}^r_nN^{rs\,*}_{nm}\widetilde{\alpha}^s_m\right.$$
$$\left. + \ln\frac{3^3}{2^4}\sum_{r=1}^{3}N_r\right)G_{\mu,\nu}(p_3)\alpha^{3\mu}_{-1}\widetilde{\alpha}^{3\mu}_{-1}|0_1\rangle|0_2\rangle|0_3\rangle$$

where $\sum_{r=1}^{3}N_r = 0+0+1 = 1$. Expanding the exponential in a Taylor series, the zero-order terms vanish because the operators α_{-1} or $\widetilde{\alpha}_{-1}$ annihilate the left vacuum. For the

† See also references [8–12] given in [14].

same reason, the only terms that contribute are the ones that contain one α_{-1} operator or one $\widetilde{\alpha}_{-1}$, since for example commuting α_{-1} with one α_n of any of the terms quadratic in α_m's say, gets rid of the creation operator leaving the annihilation operators free to kill the right vacuum. Using the results of (I) one obtains

$$
\begin{aligned}
A_3(T,T,G) &= \langle 0_{123}| \left(\frac{3^3}{2^4}\right) \exp\left(\frac{-\mathrm{i}\alpha_1^i}{\sqrt{2}} \boldsymbol{G}_1^{(2)\prime i,j} P^j + \frac{\mathrm{i}\widetilde{\alpha}_1^i}{\sqrt{2}} \boldsymbol{G}_1^{(2)\prime i,j} P^j\right) \\
&\quad \times \alpha_{-1}^{3\mu} \widetilde{\alpha}_{-1}^{3\mu} |0_1\rangle |0_2\rangle |0_3\rangle G_{\mu,\nu}(p_3) \\
&= \left(\frac{3^3}{2^4}\langle 0_{123}|\right) \left(\frac{1}{2} \boldsymbol{G}_1^{(2)\prime\, 3,j} P^{j\mu} \boldsymbol{G}_1^{(2)\prime 3,k} P^{k\nu}\right) |0_1\rangle |0_2\rangle |0_3\rangle G_{\mu,\nu}(p_3).
\end{aligned}
$$

Using the value of $\boldsymbol{G}_1^{(2)\prime\, i,j}$ given in (I), (with $U_1 = 2/3$) we arrive at the final expression for the amplitude, namely

$$
\begin{aligned}
A_3(T,T,G) &= \frac{1}{16}(P_1 - P_2)^\mu (P_1 - P_2)^\nu G_{\mu,\nu}(P_3) \\
&= \frac{1}{4}\left(\frac{P_1}{2} - \frac{P_2}{2}\right)^\mu \left(\frac{P_1}{2} - \frac{P_2}{2}\right)^\nu G_{\mu,\nu}(P_3)
\end{aligned}
$$

where we have made use of the fact that $G_\mu^\mu = 0$. This should be compared with the standard result

$$A^{\mathrm{closed}}(P_3) = A^{\mathrm{open}}(P_3/2) A^{\mathrm{open}}(P_3/2)^*$$

where $A^{\mathrm{open}}(P_3)$ is the open string (tachyon, tachyon, vector) amplitude which is given by

$$A^{\mathrm{open}}(P_3) = \tfrac{1}{2}(P_1 - P_2)^\mu A_\mu(P_3)$$

where P_1 and P_2 are the momenta of the tachyons and $A_\mu(P_3)$ represents the vector particle (one has to make the identification $G_{\mu,\nu} \equiv A_\mu A_\nu$ at the end).

It is satisfying to see that the HS vertices calculated using the HS language do indeed satisfy the GGR theorem and that the three-amplitudes calculated here agree with the standard results obtained in the literature. This indicates that the HS approach to string field theory is on a strong footing.

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